Mathematics Graded Course of Study

GRADES K - 12

HURON CITY SCHOOLS

2017

Huron City Schools 712 Cleveland Road E. Huron, Ohio 44839

Graded Course of Study for Mathematics Grades K - 12

> Dennis Muratori Superintendent

> > 2017

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INTRODUCTION

This document represents our answer to the charge placed upon us by the Ohio Department of Education in developing a graded Mathematics Course of Study.

The Huron City School District's Grades K-High School Graded Course of Study for Mathematics communicates with staff, students, parents, and community the overall learning targets and performance expectations of the program. This Graded Course of Study incorporates Huron City Schools' Mission and Vision, and the Ohio Department of Education Revised Learning Standards adopted in 2017. This document is intended to outline learning expectations and provide a tool for documenting and tracking an individual's development, progress, and achievement over the course of the student's school career.

This revision places a greater emphasis on College and Career Readiness as well as the development of 21st century skills of collaboration, creativity, critical thinking, and communication.

MISSION

The mission of Huron Schools is to engage, equip and empower students as global citizens to positively impact the future.

VISION

Huron Schools:

- A commitment to innovation
- A passion for excellence
- A catalyst for growth

CORE VALUES

As we strive to attain our vision and work to accomplish our mission, we value:

- · Highly qualified and compassionate faculty and staff.
- · Life-long learning, scholarship, leadership and service.
- Commitment to excellence in academics, athletics and arts.
- The full potential of each high school staff member to instruct in both the high school and higher education realms through strong partnerships with institutions of higher learning.
- A variety of student experiences preparing them for a competitive post-graduate life.
- Engagement and collaboration with all stakeholders parents, community partners and staff.
- A culture that breeds forward thinking leaders and independent thought.
- Individual interactions that build connections and commitments.
- Courage, determination and integrity.
- · Celebrating all students at all levels.
- Diversity of cultures, experiences and opinions.
- Pride, hard work and strong credentials.
- Partnerships with area businesses and city and township agencies resulting in the highest level of service without duplication and with the effective use of taxpayer dollars.
- Technology is a tool, not a teacher.
- Leaders who make students the focus of every decision.
- A safe, friendly and challenging environment.
- Preparing students to successfully adapt to continual change.

KINDERGARTEN

Counting and Cardinality

Know number names and the count sequence.

- K.CC.1 Count to 100 by ones and by tens.
- K.CC.2 Count forward within 100 beginning from any given number other than 1.
- K.CC.3 Write numerals from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

- K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality using a variety of objects including pennies.
 - a. When counting objects, establish a one-to-one relationship by saying the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
 - b. Understand that the last number name said tells the number of objects counted and that the number of objects is the same regardless of the arrangement or the order in which counted.
 - c. Understand that each successive number name refers to a quantity that is one larger.
- K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

Compare numbers.

- K.CC.6 Orally identify (without using inequality symbols) whether the number of objects in one group is greater/more than, less/fewer than, or the same as the number of objects in another group, not to exceed 10 objects in each group.
- K.CC.7 Compare (without using inequality symbols) two numbers between 0 and 10 when presented as written numerals.

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

- K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds such as claps, acting out situations, verbal explanations, expressions, or equations. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)
- K.OA.2 Solve addition and subtraction problems (written or oral), and add and subtract within 10 by using objects or drawings to represent the problem.
- K.OA.3 Decompose numbers and record compositions for numbers less than or equal to 10 into pairs in more than one way by using objects and, when appropriate, drawings or equations.
- K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or, when appropriate, an equation.
- K.OA.5 Fluently add and subtract within 5.

Numbers and Operations in Base Ten

Work with numbers 11–19 to gain foundations for place value.

K.NBT.1 Compose and decompose numbers from 11 to 19 into a group of ten ones and some further ones by using objects and, when appropriate, drawings or equations; understand that these numbers are composed of a group of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data

Identify, describe, and compare measurable attributes.

- K.MD.1 Identify and describe measurable attributes (length, weight, and height) of a single object using vocabulary terms such as long/short, heavy/light, or tall/short.
- K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe, one child as taller/shorter.

Classify objects and count the number of objects in each category.

K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. The number of objects in each category should be less than or equal to ten. Counting and sorting coins should be limited to pennies.

Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

- K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
- K.G.2 Correctly name shapes regardless of their orientations or overall size.
- K.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Describe, compare, create, and compose shapes.

- K.G.4 Describe and compare two- or three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, commonalities, parts, and other attributes.
- K.G.5 Model shapes in the world by building shapes from components, e.g., sticks and clay balls, and drawing shapes.
- K.G.6 Combine simple shapes to form larger shapes.

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FIRST GRADE

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
- 1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Understand and apply properties of operations and the relationship between addition and subtraction.

- 1.OA.3 Apply properties of operations as strategies to add and subtract. For example, if 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (Commutative Property of Addition); to add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (Associative Property of Addition). Students need not use formal terms for these properties.
- 1.OA.4 Understand subtraction as an unknown addend problem. For example, subtract 10 8 by finding the number that makes 10 when added to 8.

Add and subtract within 20.

- 1.OA.5 Relate counting to addition and subtraction, e.g., by counting on 2 to add 2.
- 1.OA.6 Add and subtract within 20, demonstrating fluency with various strategies for addition and subtraction within 10. Strategies may include counting on; making ten, e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14; decomposing a number leading to a ten, e.g., 13 4 = 13 3 1 = 10 1 = 9; using the relationship between addition and subtraction, e.g., knowing that 8 + 4 = 12, one knows 12 8 = 4; and creating equivalent but easier or known sums, e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13.

Work with addition and subtraction equations.

- 1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.
- 1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 + \ddot{y} = 11$, $5 = \ddot{y} = 3$, $6 + 6 = \ddot{y}$.

Numbers and Operations in Base Ten

Extend the counting sequence.

1.NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

- 1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones called a "ten;" the numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones; and the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
- 1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

- 1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; record the strategy with a written numerical method (drawings and, when appropriate, equations) and explain the reasoning used. Understand that when adding two-digit numbers, tens are added to tens; ones are added to ones; and sometimes it is necessary to compose a ten.
- 1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
- 1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Measurement and Data

Measure lengths indirectly and by iterating length units.

- 1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.
- 1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

Work with time and money.

- 1.MD.3 Work with time and money.
 - a. Tell and write time in hours and half-hours using analog and digital clocks.
 - b. Identify pennies and dimes by name and value.

Represent and interpret data.

1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry

Reason with shapes and their attributes.

- 1.G.1 Distinguish between defining attributes, e.g., triangles are closed and three-sided, versus nondefining attributes, e.g., color, orientation, overall size; build and draw shapes to possess defining attributes.
- 1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quartercircles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. Students do not need to learn formal names such as "right rectangular prism."
- 1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths*, and *quarters*, and use the phrases *half of, fourth of*, and *quarter of*. Describe the whole as two of or four of the shares in real-world contexts. Understand for these examples that decomposing into more equal shares creates smaller shares.

SECOND GRADE

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Add and subtract within 20.

2.OA.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. [Mental Strategies]

Work with equal groups of objects to gain foundations for multiplication.

- 2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
- 2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Numbers and Operations in Base Ten

Understand place value.

- 2.NBT.1 Understand that the three digits of a three digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
 - a. 100 can be thought of as a bundle of ten tens called a "hundred."
 - b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
- 2.NBT.2 Count forward and backward within 1000 by ones, tens, and hundreds starting at any number; skip-count by 5s starting at any multiple of 5.
- 2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, expanded form, and equivalent representations, e.g., 716 is 700 + 10 + 6, or 6 + 700 + 10, or 6 ones and 71 tens, etc.

- 2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

 Use place value understanding and properties of operations to add and subtract.
- 2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
- 2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.
- 2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; record the strategy with a written numerical method (drawings and, when appropriate, equations) and explain the reasoning used. Understand that in adding or subtracting three-digit numbers, hundreds are added or subtracted from hundreds, tens are added or subtracted from tens, ones are added or subtracted from ones; and sometimes it is necessary to compose or decompose tens or hundreds.
- 2.NBT.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
- 2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. Explanations may be supported by drawings or objects.

Measurement and Data

Measure and estimate lengths in standard units.

- 2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
- 2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
- 2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.
- 2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

- 2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same whole number units, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)
- 2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2,..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

- 2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
- 2.MD.8 Solve problems with money.
 - a. Identify nickels and quarters by name and value.
 - b. Find the value of a collection of quarters, dimes, nickels, and pennies.
 - c. Solve word problems by adding and subtracting within 100, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the \$ and \$\mathcal{C}\$ symbols appropriately (not including decimal notation).

Represent and interpret data.

- 2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object. Show the measurements by creating a line plot, where the horizontal scale is marked off in whole-number units.
- 2.MD.10 Organize, represent, and interpret data with up to four categories; complete picture graphs when single-unit scales are provided; complete bar graphs when single-unit scales are provided; solve simple put-together, take-apart, and compare problems in a graph.

Geometry

2.G.1 Reason with shapes and their attributes.

- 2.G.1 Recognize and identify triangles, quadrilaterals, pentagons, and hexagons based on the number of sides or vertices. Recognize and identify cubes, rectangular prisms, cones, and cylinders.
- 2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
- 2.G.3 Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, or fourths and quarters, and use the phrases half of, third of, or fourth of and quarter of. Describe the whole as two halves, three thirds, or four fourths in real-world contexts. Recognize that equal shares of identical wholes need not have the same shape.

THIRD GRADE

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

- 3.OA.1 Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. (Note: These standards are written with the convention that a x b means a groups of b objects each; however, because of the commutative property, students may also interpret 5 x 7 as the total number of objects in 7 groups of 5 objects each).
- 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.
- 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Glossary, Table 2. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)
- 3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times \ddot{y} = 48$, $5 = \ddot{y} \div 3$, $6 \times 6 = \ddot{y}$.

Understand properties of multiplication and the relationship between multiplication and division.

- 3.OA.5 Apply properties of operations as strategies to multiply and divide. For example, if 6 × 4 = 24 is known, then 4 × 6 = 24 is also known (Commutative Property of Multiplication); 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30 (Associative Property of Multiplication); knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (Distributive Property). Students need not use formal terms for these properties.
- 3.OA.6 Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

Multiply and divide within 100.

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division, e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8 or properties of operations. Limit to division without remainders. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

- 3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter or a symbol, which stands for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole-number answers. Students may use parentheses for clarification since algebraic order of operations is not expected.
- 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Numbers and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of strategies and algorithms may be used.

- 3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.
- 3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
- 3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90, e.g., 9 × 80, 5 × 60 using strategies based on place value and properties of operations.

Numbers and Operations- Fractions

Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

- 3.NF.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.
- 3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.
 - a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
 - b. Represent a fraction *alb* (which may be greater than 1) on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size *alb* and that its endpoint locates the number *alb* on the number line.
- 3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
 - a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.
 - b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
 - c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.
 - d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data

Solve problems involving money, measurement, and estimation of intervals of time, liquid volumes, and masses of objects.

- 3.MD.1 Work with time and money.
 - a. Tell and write time to the nearest minute. Measure time intervals in minutes (within 90 minutes). Solve real-world problems involving addition and subtraction of time intervals (elapsed time) in minutes, e.g., by representing the problem on a number line diagram or clock.
 - b. Solve word problems by adding and subtracting within 100, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the \$ and © symbol appropriately (not including decimal notation).
- 3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. Add, subtract, multiply, or divide whole numbers to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems involving notions of "times as much";

Represent and interpret data.

- 3.MD.3 Create scaled picture graphs to represent a data set with several categories. Create scaled bar graphs to represent a data set with several categories. Solve two step "how many more" and "how many less" problems using information presented in the scaled graphs. For example, create a bar graph in which each square in the bar graph might represent 5 pets, then determine how many more/less in two given categories.
- 3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by creating a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

- 3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.
 - a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
 - b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
- 3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
- 3.MD.7 Relate area to the operations of multiplication and addition.
 - a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
 - b. Multiply side lengths to find areas of rectangles with whole- number side lengths in the context of solving real world and mathematical problems, and represent whole number products as rectangular areas in mathematical reasoning.
 - c. Use tiling to show in a concrete case that the area of a rectangle with wholenumber side lengths a and b + c is the sum of $a \times b$ and $a \times c$ (represent the distributive property with visual models including an area model).
 - d. Recognize area as additive. Find the area of figures composed of rectangles by decomposing into nonoverlapping rectangles and adding the areas of the nonoverlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry

Reason with shapes and their attributes.

- 3.G.1 Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles).
- 3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.

FOURTH GRADE

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

- 4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
- 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. See Glossary, Table 2. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)
- 4.OA.3 Solve multistep word problems posed with whole numbers and having wholenumber answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Numbers and Operations in Base Ten

Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.

- 4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right by applying concepts of place value, multiplication, or division.
- 4.NBT.2 Read and write multi-digit whole numbers using standard form, word form, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.</p>
- 4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place through 1,000,000.

Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers less than or equal to 1,000,000.

- 4.NBT.4 Fluently add and subtract multi-digit whole numbers using a standard algorithm.
- 4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
- 4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Numbers and Operations-Fractions

Extend understanding of fraction equivalence and ordering limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

- 4.NF.1 Explain why a fraction a /b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
- 4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified.)

- 4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
 - a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
 - b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 : 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
 - c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
 - d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

- 4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
 - a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$, or 5/4 = 1/4 + 1/4 + 1/4 + 1/4.
 - b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)
 - c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Understand decimal notation for fractions, and compare decimal fractions limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified.)

- 4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general, but addition and subtraction with unlike denominators in general is not a requirement at this grade.
- 4.NF.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.
- 4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

- 4.MD.1 Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. For example, express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a two-column table of meters and centimeters includes the number pairs 1 and 100, 2 and 200, 3 and 300,...
- 4.MD.2 Solve real-world problems involving money, time, and metric measurement.
 - Using models, add and subtract money and express the answer in decimal notation.
 - b. Using number line diagrams, clocks, or other models, add and subtract intervals of time in hours and minutes.
 - c. Add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects.
- 4.MD.3 Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.

Represent and interpret data.

4.MD.4 Display and interpret data in graphs (picture graphs, bar graphs, and line plots) to solve problems using numbers and operations for this grade.

Geometric measurement: understand concepts of angle and measure angles.

- 4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.
 - a. Understand an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
 - b. Understand an angle that turns through n one degree angles is said to have an angle measure of n degrees.
- 4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
- 4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

- 4.G.1 Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
- 4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.

FIFTH GRADE

Algebraic Thinking

Write and interpret numerical expressions.

- 5.OA.1 Use parentheses in numerical expressions, and evaluate expressions with this symbol. Formal use of algebraic order of operations is not necessary.
- 5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product.

Analyze patterns and relationships.

5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Numbers and Operations in Base Ten

Understand the place value system.

- 5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
- 5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use wholenumber exponents to denote powers of 10.
- 5.NBT.3 Read, write, and compare decimals to thousandths.
 - a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.
 - b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
- 5.NBT.4 Use place value understanding to round decimals to any place, millions through hundredths.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

- 5.NBT.5 Fluently multiply multi-digit whole numbers using a standard algorithm. Standard 2010 Standard 2016 Proposed Standard
- 5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
- 5.NBT.7 Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, or multiplication and division; relate the strategy to a written method and explain the reasoning used.
 - a. Add and subtract decimals, including decimals with whole numbers, (whole numbers through the hundreds place and decimals through the hundredths place).
 - b. Multiply whole numbers by decimals (whole numbers through the hundreds place and decimals through the hundredths place).
 - c. Divide whole numbers by decimals and decimals by whole numbers (whole numbers through the tens place and decimals less than one through the hundredths place using numbers whose division can be readily modeled). For example, 0.75 divided by 5, 18 divided by 0.6, or 0.9 divided by 3.

Numbers and Operations—Fractions

Use equivalent fractions as a strategy to add and subtract fractions (Fractions need not be simplified.)

- 5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, use visual models and properties of operations to show 2/3 + 5/4 = 8/12 + 15/12 = 23/12. In general, a/b + c/d = (a/b x d/d) + (c/d x b/b) = (ad + bc)/bd.
- 5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Fractions need not be simplified.)

- 5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
- 5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
 - a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts, equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)
 - b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
- 5.NF.5 Interpret multiplication as scaling (resizing).
 - a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
 - b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
- 5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

- 5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade.
 - a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.
 - b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.
 - c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3- cup servings are in 2 cups of raisins?

Measurement and Data

Convert like measurement units within a given measurement system.

5.MD.1 Know relative sizes of these U.S. customary measurement units: pounds, ounces, miles, yards, feet, inches, gallons, quarts, pints, cups, fluid ounces, hours, minutes, and seconds. Convert between pounds and ounces; miles and feet; yards, feet, and inches; gallons, quarts, pints, cups, and fluid ounces; hours, minutes, and seconds in solving multi-step, real-world problems.

Represent and interpret data.

5.MD.2 Display and interpret data in graphs (picture graphs, bar graphs, and line plots) to solve problems using numbers and operations for this grade, e.g., including U.S. customary units in fractions 1/2, 1/4, 1/8, or decimals.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

- 5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
 - a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
 - b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
- 5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- 5.MD.5 Relate volume to the operations of multiplication and addition and solve realworld and mathematical problems involving volume.
 - a. Find the volume of a right rectangular prism with whole number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the Associative Property of Multiplication.
 - b. Apply the formulas $V = I \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Geometry

Graph points on the coordinate plane to solve real-world and mathematical problems.

- 5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond, e.g., *x* -axis and *x* -coordinate, *y* -axis and *y* coordinate.
- 5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

- 5.G.3 Identify and describe commonalities and differences between types of triangles based on angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral, and scalene triangles).
- 5.G.4 Identify and describe commonalities and differences between types of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares, rectangles, parallelograms, trapezoids, and rhombuses.

SIXTH GRADE

Ratios and Proportional Relationships

6.RP.1 Understand ratio concepts and use ratio reasoning to solve problems.

- 6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."
- 6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger."
- 6.RP.3 Use ratio and rate reasoning to solve real world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
 - a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
 - b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
 - c. Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means 30/100 times the quantity; solve problems involving finding the whole, given a part and the percent.
 - d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share ½ lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Compute fluently with multi-digit numbers and find common factors and multiples.

- 6.NS.2 Fluently divide multi-digit numbers using a standard algorithm.
- 6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.
- 6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge; use positive and negative numbers to represent quantities in real world contexts, explaining the meaning of 0 in each situation.

- 6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
 - a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.
 - b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
 - c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

- 6.NS.7 Understand ordering and absolute value of rational numbers.
 - a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.
 - b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write -3 °C > -7 °C to express the fact that -3 °C is warmer than -7 °C.
 - c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.
 - d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.
- 6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

- 6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.
- 6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.
 - a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 y.
 - b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.
 - c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.
- 6.EE.3 Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.
- 6.EE.4 Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them. For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.

Reason about and solve one-variable equations and inequalities,

- 6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- 6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers.
- 6.EE.8 Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

Geometry

Solve real-world and mathematical problems involving area, surface area, and volume.

- 6.G.1 Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems.
- 6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = lwh and V = B h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
- 6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- 6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real world and mathematical problems.

Statistics and Probability

Develop understanding of statistical problem solving.

- 6.SP.1 Develop statistical reasoning by using the GAISE model:
 - a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because of the variability in students' ages.
 - b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question.
 - c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group.
 - d. Interpret Results: Draw logical conclusions from the data based on the original question.

Develop understanding of statistical variability.

- 6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
- 6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number

Summarize and describe distributions.

- 6.SP.4 Display numerical data in plots on a number line, including dot plots (line plots), histograms, and box plots.
- 6.SP.5 Summarize numerical data sets in relation to their context.
 - a. Report the number of observations.
 - b. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.
 - c. Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range) as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution.
 - d. Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered.