

8-7 Study Guide and Intervention

Solving $ax^2 + bx + c = 0$

Factor $ax^2 + bx + c$ To factor a trinomial of the form $ax^2 + bx + c$, find two integers, m and p whose product is equal to ac and whose sum is equal to b . If there are no integers that satisfy these requirements, the polynomial is called a **prime polynomial**.

Example 1 Factor $2x^2 + 15x + 18$.

In this example, $a = 2$, $b = 15$, and $c = 18$. You need to find two numbers that have a sum of 15 and a product of $2 \cdot 18$ or 36. Make a list of the factors of 36 and look for the pair of factors with a sum of 15.

| Factors of 36 | Sum of Factors |
|---------------|----------------|
| 1, 36 | 37 |
| 2, 18 | 20 |
| 3, 12 | 15 |

Use the pattern $ax^2 + mx + px + c$, with $a = 2$, $m = 3$, $p = 12$, and $c = 18$.

$$\begin{aligned} 2x^2 + 15x + 18 &= 2x^2 + 3x + 12x + 18 \\ &= (2x^2 + 3x) + (12x + 18) \\ &= x(2x + 3) + 6(2x + 3) \\ &= (x + 6)(2x + 3) \end{aligned}$$

Therefore, $2x^2 + 15x + 18 = (x + 6)(2x + 3)$.

Example 2 Factor $3x^2 - 3x - 18$.

Note that the GCF of the terms $3x^2$, $3x$, and 18 is 3. First factor out this GCF.

$$3x^2 - 3x - 18 = 3(x^2 - x - 6)$$

Now factor $x^2 - x - 6$. Since $a = 1$, find the two factors of -6 with a sum of -1 .

| Factors of -6 | Sum of Factors |
|-----------------|----------------|
| 1, -6 | -5 |
| $-1, 6$ | 5 |
| $-2, 3$ | 1 |
| 2, -3 | -1 |

Now use the pattern $(x + m)(x + p)$ with $m = 2$ and $p = -3$.

$$x^2 - x - 6 = (x + 2)(x - 3)$$

The complete factorization is

$$3x^2 - 3x - 18 = 3(x + 2)(x - 3)$$

Exercises

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1. $2x^2 - 3x - 2$

2. $3m^2 - 8m - 3$

3. $16r^2 - 8r + 1$

4. $6x^2 + 5x - 6$

5. $3x^2 + 2x - 8$

6. $18x^2 - 27x - 5$

7. $2a^2 + 5a + 3$

8. $18y^2 + 9y - 5$

9. $-4t^2 + 19t - 21$

10. $8x^2 - 4x - 24$

11. $28p^2 + 60p - 25$

12. $48x^2 + 22x - 15$

13. $3y^2 - 6y - 24$

14. $4x^2 + 26x - 48$

15. $8m^2 - 44m + 48$

16. $6x^2 - 7x + 18$

17. $2a^2 - 14a + 18$

18. $18 + 11y + 2y^2$